Substituting for $\tan y$ its series, and integrating, we get

$$\int\frac{\tan y dy}{y}=\frac{2^2(2^2-1)}{2!}B_1 y+\frac{2^4(2^4-1)}{4!}B_3\frac{y^3}{3}+\frac{2^6(2^6-1)}{6!}B_5\frac{y^5}{5},$$

where B_1, B_3, B_5, ... are Bernoulli's numbers.

Therefore the given integral is

$$x\log.\tan^{-1}x-\left[\frac{2^2(2^2-1)}{2!}B_1\tan^{-1}x+\frac{2^4(2^4-1)}{4!}B_3\frac{(\tan^{-1}x)^3}{3}+\ldots\right].$$

Also solved by G. B. M. Zerr, V, M. Spunar, and J. E. Sanders.

273. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

On one side of a circular pond a feet in radius is a duck. On the diametrically opposite side of the pond is a dog. Both begin to swim at the same time, the duck swimming around the circumference of the pond at the rate of m feet a minute, the dog swimming directly towards the duck at the rate of n feet per minute. How far will the dog swim in overtaking the duck?

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

If the line joining the positions of the dog and the duck is tangent to the dog's path, the following is the solution.

Let A, B be the starting points of dog and duck; O, the center of the pond; P, R, corresponding positions of the dog and duck. Then the tangent t, from P must pass through R. Let the duck swim to S and the dog to Q. Draw OF, PH perpendicular to AB, PI, QD perpendicular to OF, QC perpendicular to PI, and RE perpendicular to QS, produced.

Let $AB=2a$, $AP=\sigma$, $\angle RAB=\phi$, $OD=x$, $n/m=b$. Then $PR=d\sigma$, $\angle RAS=d\phi$, $DI=QC=dx$.

Now $\sigma=2ab\phi$; $\therefore d\sigma/2ad\phi=b$.

In the limit the triangles PQC and RES are similar.

$\therefore d\sigma : dx=2ad\phi : ES$. $\therefore ES=(2ad\phi/d\sigma)dx=dx/b$.

The tangent t has negative increments at both ends (PQ and ES).

$\therefore dt=-d\sigma-dx/b$, or $t=C-\sigma-x/b$. When $t=2a$, $\sigma=0$, $x=0$.

$\therefore C=2a$. $\therefore t=2a-2ab\phi-x/b$. When $t=0$, $x=2a\sin\phi\cos\phi$.

$\therefore b=b^2\phi+\sin\phi\cos\phi=(b^2+1)\phi-\frac{2}{3}\phi^3+\frac{2}{15}\phi^5-\frac{4}{315}\phi^7+\ldots$

$$Q=\frac{b}{b^2+1}+\frac{2b^4}{3(b^2+1)^4}+\frac{2b^5(9-b^2)}{15(b^2+1)^7}+\frac{4(225+b^4-54b^2)b^7}{315(b^2+1)^{10}}+\ldots$$

by reversion of series.

b. If the dog always keeps on the line joining his starting point to the duck's position, the solution becomes quite simple.

Then $us=2a\psi$ is the intrinsic equation to the dog's path.

$$\therefore \frac{ds}{d\psi}=\frac{2a}{u}=\sqrt{r^2+\left(\frac{dr}{d\psi}\right)^2}. \therefore d\psi=\frac{dr}{\sqrt{\frac{4a^2}{u^2}-r^2}}, \text{ and } \psi=\sin^{-1}\frac{ur}{2a}.$$

When $r=2a\cos\psi$ the dog catches the duck.

$\therefore \psi=\sin^{-1}(u\cos\psi)$, or $\tan\psi=u$, and $\psi=\tan^{-1}u$.

$\therefore us=2a\tan^{-1}u$, or $s=(2a/u)\tan^{-1}u=(2an/m)\tan^{-1}(m/n)$ is the distance the dog swims to overtake the duck.

MECHANICS.

225. Proposed by W. A. BALDWIN, Springfield, Mo.

Find, *by means of polar coordinates,* the moment of inertia about the origin of the area between the parabola $ay=2(a^2-x^2)$, the circle $x^2+y^2=a^2$, and the axis of Y.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

The polar equation to the parabola is

$$r=\frac{a[\sqrt{(1+15\cos^2\theta)}-\sin\theta]}{4\cos^2\theta}=r_1.$$

The polar equation to the circle is $r=a$.

$\therefore m$, moment of inertia required,

$$=\int_{\pi/6}^{\pi/2}\int_a^{r_1} r^3 d\theta\, dr=\frac{a^4}{1024}\int_{\pi/6}^{\pi/2}[8\sec^8\theta+112\sec^6\theta+136\sec^4\theta-256$$

$$-8\sin\theta\sec^6\theta(7+\sec^2\theta)\sqrt{(1+15\cos^2\theta}]d\theta=\frac{a^4}{4}\left(\frac{883\sqrt{3}}{280}-\frac{\pi}{3}\right).$$

This is problem 7, p. 350, Osborne's *Differential and Integral Calculus.* Osborne gives as the result, $(\frac{52}{35}-\frac{1}{8}\pi)a^4$. Professor Zerr's result is correct. ED. F.